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Analysis of Fuzzy Tandem Queues By Flexible α-Cuts Method

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Abstract-Fuzzy tandem queues is a gadget outstretched in stacks of real world applications such as machines, production models and service stations. This critique amalgamates a two-station fuzzy tandem queues opting the approach called "flexible α-cuts method". This method procures the characteristics of fuzzy tandem queues based on alpha-cuts and interval arithmetic. The efficacy of the proposed approach is validated with numerical illustration.

Keywords: Tandem queues, triangular fuzzy numbers, α-cuts and interval arithmetic, flexible α-cuts method and queuing theory.

1. INTRODUCTION

Queuing theory as an operations management technique furnishes extensive applicability in management enterprises, production units and system analysis based on specific queue discipline. In a predestinate universe, fuzziness contributes strong affinity in bounded uncertainty, depicts vagueness and universally generates its feasibility in all environments.

Fuzzy tandem queues are the most coherent non-trivial network of queues. In this paper, we consider single channel queuing system with two series station. Customers enter first queue according to a poisson process of fuzzy arrival rate λ and after completion of service directly enter the second queue with fuzzy service rate $\tilde{\mu}$.

A strict mathematical framework of fuzziness was introduced by Lofti A. Zadeh (1978). The concept of tandem queues has been applied by various researchers in different forms. Y. Li, X. Cai, F. Tu and X. Shao [7], J. Resing and L. Ormeci [14], K.H. Chang and W.F. Chen [4], A. Gomez – Corral [5], W. Bohm [1], K. Nakade [11] have discussed. The crisp tandem queues when stretched to fuzzy tandem queues have progressive utilization in queueing models. Shin-Pin, Chen [16] derived a mathematical programming approach to fuzzy tandem queues, A. Nagoor Gani and W. Ritha [10] constructed fuzzy tandem queues, Runtong Zhang and Yannis A. Phillis [15] studied fuzzy control of arrivals to tandem queues with two stations. J.P. Mukeba K[9] has analyzed that fuzzy queue characteristics can be constructed by a new process called "flexible α-cuts method". We extend this idea by taking fuzzy tandem queues and applying flexible α-cuts method with parametric non-linear programming approach to explore its flexibility and effectiveness.

2. BASIC PRELIMS

Triangular Fuzzy Number (Tfn)

A fuzzy number $\tilde{T}$ is said to be a triangular fuzzy number iff there exists three real numbers $t_1 < t_2 < t_3$ such that

$$M_{\tilde{T}}(x) = \begin{cases} 0, \ x < t_1 \\ \dfrac{x - t_1}{t_2 - t_1}, \ t_1 \le x \le t_2 \\ \dfrac{t_3 - x}{t_3 - t_2}, \ t_2 \le x \le t_3 \\ 0, \quad x > t_3 \end{cases}$$

3. DEFUZZIFICATION

Let $\tilde{T} = (t_1, t_2, t_3)$ be a TFN, then its correlated crisp number is

$$\tilde{T} = \frac{t_1 + 2t_2 + t_3}{4}$$

Definition Of α-Cut

Let $\tilde{F}$ be a fuzzy subset in the universe U. The alpha-cut $\tilde{F}_\propto$, support ($\tilde{F}$), height ($\tilde{F}$) and the core ($\tilde{F}$) are ordinary sets in [0, 1] defined as:

$$\tilde{F}_\propto = \{x \in U; \eta_{\tilde{F}}(x) \ge \alpha\}$$

$$\text{Support } (\tilde{F}) = \{x \in U; \eta_{\tilde{F}}(x) > 0\}$$

$$\text{Height } (\tilde{F}) = \max \{\eta_{\tilde{F}}(x) \mid x \in U\}$$

$$\text{Core } (\tilde{F}) = \{x \in U \mid \eta_{\tilde{F}}(x) = 1\}$$

and its membership function is

$$\eta_{\tilde{F}}(x) = \operatorname*{Sup}_{\alpha \in [0,1]} \min\{\alpha, \eta_{\tilde{F}_\alpha}(x)\}$$

where

$$\eta_{\tilde{F}_\alpha}(x) = \begin{cases} 1, & \text{if } x \in \tilde{A}_\alpha \\ 0, & \text{otherwise} \end{cases}$$

4. REMARK

The α-cuts arithmetic is applied for defuzzification, interval arithmetic for performing classical arithmetic and characteristic function of α-cuts is needed for fuzzification.

Fuzzy Arithmetic Based On α-Cuts And Interval Arithmetic

(i) α-cuts arithmetic

Let $\tilde{T}$ and $\tilde{S}$ be two fuzzy numbers with fuzzy arithmetic operation through their α-cuts in [0,1]. If $\tilde{T}_\alpha = [T^L(\alpha), T^U(\alpha)]$ and $\tilde{S}_\alpha = [S^L(\alpha), S^U(\alpha)]$ are the α-cuts of $\tilde{T}$ and $\tilde{S}$ then the operations are:

(i) $[\tilde{T} \oplus \tilde{S}]_\alpha = \tilde{T}_\alpha + \tilde{S}_\alpha = [T^L(\alpha), T^U(\alpha)] + [S^L(\alpha), S^U(\alpha)]$

(ii) $[\tilde{T} \Theta \tilde{S}]_\alpha = \tilde{T}_\alpha - \tilde{S}_\alpha = [T^L(\alpha), T^U(\alpha)] - [S^L(\alpha), S^U(\alpha)]$

(iii) $[\tilde{T} \otimes \tilde{S}]_a = \tilde{T}_a . \tilde{S}_a = [T^L(a), T^U(a)].[S^L(a), S^U(a)]$

(iv) $[\tilde{T} \oslash \tilde{S}]_a = \dfrac{\tilde{T}_a}{\tilde{S}_a} = \dfrac{[T^L(a), T^U(a)]}{[S^L(a), S^U(a)]}$

(ii) Interval arithmetic

Let $[p_1, q_1]$ and $[p_2, q_2]$ be two closed and bounded real intervals.

If $*$ denotes. addition, subtraction, multiplication or division, then $[p_1, q_1] * [p_2, q_2] = [\alpha, \beta]$ where $[\alpha, \beta] = \{p * q : p_1 \le p \le q, p_2 \le q \le q_2\}$ with its operations based on the above equation is:

(i) $[p_1, q_1] + [p_2, q_2] = [p_1 + p_2, q_1 + q_2]$

(ii) $[p_1, q_1] - [p_2, q_2] = [p_1 - p_2, q_1 - q_2]$

(iii) $[p_1, q_1].[p_2, q_2] = \begin{matrix} [\min\{p_1p_2, p_1q_2, q_1p_2, q_1q_2\}, \\ \max\{p_1p_2, p_1q_2, q_1p_2, q_1q_2\}] \end{matrix}$

(iv) $\dfrac{[p_1, q_1]}{[p_2, q_2]} = \left[\min\left\{\dfrac{p_1}{p_2}, \dfrac{p_1}{q_2}, \dfrac{q_1}{p_2}, \dfrac{q_1}{q_2}\right\}, \max\left\{\dfrac{p_1}{p_2}, \dfrac{p_1}{q_2}, \dfrac{q_1}{p_2}, \dfrac{q_1}{q_2}\right\}\right]$, provided $0 \notin [p_2, q_2]$

Flexible α-Cuts Method

Fuzzy model can be grounded in the flexible α-cuts method only when ordinary queue formula and fuzzy queue input parameters are known. Let us establish a characteristic ϕ of a fuzzy tandem queue with input parameters $\tilde{x}_1, \tilde{x}_2, ..., ...\tilde{x}_n$ and assume ϕ and $x_1, x_2, ..., ...x_n$ are similar characteristic and parameters in crisp model.

Define: $\phi = f(x_1, x_2, ..., ...x_n)$ where f is a real multivalued function employing basic arithmetic operations in R. By Zadeh's extension principle, the crisp characteristic ϕ is developed to the fuzzy characteristic $\tilde{\phi} = \tilde{f}(\tilde{x}_1, \tilde{x}_2, ...\tilde{x}_n)$ where $\tilde{f}$ is a fuzzy multivalued function with basic fuzzy arithmetic operations in $F(R)$.

To manipulate the fuzzy tandem queue characteristic $\tilde{\phi}$ by flexible α-cuts approach, we use the steps as follows:

(i) Resolve the α-cuts of all input parameter.

(ii) Apply α-cuts fuzzy arithmetic to fuzzy tandem queue characteristic $\tilde{\phi}$

(iii) Use interval arithmetic to obtain $\tilde{\phi}_\alpha = [\tilde{\phi}^L(\alpha), \tilde{\phi}^U(\alpha)]$ where $\tilde{\phi}^L(\alpha)$ and $\tilde{\phi}^U(\alpha)$ are real-valued functions whose reciprocals characterize the membership function $\tilde{f}$ as

$$\eta_{\tilde{\phi}}(x) = \begin{cases} (\tilde{\phi}^L)^{-1}(x), & \tilde{\phi}^L(0) \le x \le \tilde{\phi}^L(1) \\ (\tilde{\phi}^U)^{-1}(x), & \tilde{\phi}^L(1) < x \le \tilde{\phi}^U(0) \\ 0, \text{otherwise.} \end{cases}$$

(iv) Put $\alpha = 0$ in step (iii) to get the support bounds of $\tilde{f}$ and signifies that $\tilde{f}$ lies between $\tilde{\phi}^L(0)$ and $\tilde{\phi}^U(0)$. $\tilde{f}$ does not fall below the lower bound or exceed the upper bond. It's modal value is $\tilde{\phi}^L(1) = \tilde{\phi}^U(1)$, the most possible value.

5. MODEL DESCRIPTION

Consider a 2-work station tandem queuing model in which customers arrive at station-1, and after completing service at this station, the customer undergoes station 2 service with fuzzy arrival rate $\tilde{\lambda}$ and fuzzy service rate $\tilde{\mu}$. The workstation is either free or busy. Station 1 is blocked if the customer in this station completes his job before station 2 is free. Waiting between workstations is prohibited. Let free, busy and blocked states be 0, l, b. The possible states are {(0, 0), (0, 1), (1, 0), (1, 1), (b, 1)}

The steady state equations are:

(i) $\tilde{p}_{0,0}(t+h) = \tilde{p}_{00}(t)(1 - \tilde{\lambda}h) + p_{0,1}(t)\tilde{\mu}h$

(ii) $\tilde{p}_{0,1}(t+h) = \tilde{p}_{01}(t)(1 - \tilde{\mu}h - \tilde{\lambda}h) + p_{1,0}(t)\tilde{\mu}h + \tilde{p}_{1,1}(t)\tilde{\mu}h + p_{b,1}(t)\tilde{\mu}h$

(iii) $\tilde{p}_{1,1}(t+h) = \tilde{p}_{0,1}(t)(\tilde{\lambda}h) + p_{1,1}(t)(1 - 2\tilde{\mu}h)$

(iv) $\tilde{p}_{b,1}(t+h) = \tilde{p}_{1,1}(t)(\tilde{\mu}h) + \tilde{p}_{b,1}(t)(1 - \tilde{\mu}h)$

(v) $\tilde{p}_{1,0}(t+h) = \tilde{p}_{0,0}(t)(\tilde{\lambda}h) + \tilde{p}_{1,0}(t)(1 - \tilde{\mu}h) + \tilde{p}_{1,1}(t)\tilde{\mu}h$

After simplification and taking limits we have,

$$\tilde{p}_{0,1} - \tilde{\rho}\tilde{p}_{0,0} = 0$$

$$\tilde{p}_{1,0} + \tilde{p}_{b,1} - (1 - \tilde{\rho})\tilde{p}_{0,1} = 0$$

$$\tilde{p}_{1,1} + \tilde{p}_{1,0} + \tilde{\rho}\tilde{p}_{0,0} = 0$$

$$\tilde{\rho}\tilde{p}_{0,1} - 2\tilde{p}_{1,1} = 0$$

$$\tilde{p}_{1,1} - \tilde{p}_{b,1} = 0$$

we have $\tilde{p}_{00} + \tilde{p}_{01} + \tilde{p}_{10} + \tilde{p}_{11} + \tilde{p}_{b1} = 1$

The solutions are:

$$\tilde{p}_{0,0} = \frac{2\tilde{\mu}^2}{3\tilde{\lambda}^2 + 4\tilde{\mu}\lambda + 2\tilde{\mu}^2} = \frac{2}{3\tilde{\rho}^2 + 4\tilde{\rho} + 2} = \frac{2}{\tilde{B}}$$

where $\tilde{B} = 3\tilde{\rho}^2 + 4\tilde{\rho} + 2$

$$\tilde{p}_{0,1} = \tilde{\rho}\,\tilde{p}_{0,0} = \tilde{\rho}.\frac{2}{\tilde{B}} = \frac{2\tilde{\rho}}{\tilde{B}}$$

$$\tilde{p}_{1,0} = \frac{\tilde{\lambda}^2 + 2\tilde{\mu}\tilde{\lambda}}{2\tilde{\mu}^2}.\tilde{p}_{0,0} = \frac{\tilde{\rho}^2 + 2\tilde{\rho}}{\tilde{B}}$$

$$\tilde{p}_{1,1} = \tilde{p}_{b,1} = \frac{\tilde{\lambda}^2}{2\tilde{\mu}^2}\,\tilde{p}_{0,0} = \frac{\tilde{\rho}^2}{\tilde{B}}$$

(i) Average number of customers in the system:

$$\tilde{L}_S^{'} = 0\tilde{p}_{0,0} + 1(\tilde{p}_{0,1} + \tilde{p}_{1,0}) + 2(\tilde{\rho}_{1,1} + \tilde{p}_{b,1})$$

$$= \frac{5\tilde{\rho}^2 + 4\tilde{\rho}}{B}$$

(ii) Average time a customer spends in the system.

$$\tilde{W}_S^{'} = \frac{\text{Expected no.of customers in the system}}{\text{Effective arrival rate}} = \frac{\tilde{L}_s^{'}}{\tilde{\lambda}_{eff}}$$

where $\tilde{\lambda}_{eff}$ is the effective fuzzy arrival rate.

6. NUMERICAL EXAMPLE

We present the example discussed by A. Nagoor Gani and W. Ritha [10] to have comparison with their results. This example is solved using flexible α-cuts method for fuzzy tandem queue characteristics.

Consider a two series work station subassembly line regulated by a belt conveyer operation. The magnitude of the modelled commodity does not admit storing more than one unit in each station. The commodity enters to the subassembly line from another manufacturing facility corresponding to position distributing with arrival rate being a triangular fuzzy number $\tilde{\lambda} = [8,10,12]$ per hour. The service times of the two work stations follow exponential distribution with average fuzzy service rates that are triangular fuzzy number denoted as $\tilde{\mu} = [6,12,18]$. All succeeding items cannot enter the assembly line directly are diverted to other subassembly line.

$$\tilde{\lambda} = [8,10,12] \text{ and } \tilde{\mu} = [6,12,18]$$

The α-cuts of $\tilde{\lambda}$ and $\tilde{\mu}$ are:

$$\tilde{\lambda}_\alpha = [2\alpha + 8, 12 - 2\alpha]$$

$$\tilde{\mu}_\alpha = [6 + 6\alpha, 18 - 6\alpha]$$

Utilisation factor:

$$\tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}} = \frac{[2\alpha + 8, 12 - 2\alpha]}{[6 + 6\alpha, 18 - 6\alpha]}$$

By interval arithmetic,

$$\tilde{\rho} = [\min F_1(\alpha), \max F_1(\alpha)]$$

where min $F_1(\alpha)$ and max $F_1(\alpha)$ are solutions of the two parametric non-linear programs (PNLP):

$$\begin{cases} \text{where } \min F_1(\alpha) = ? \\ \min F_1(\alpha) = \min\{ f_{11}(\alpha), f_{12}(\alpha), f_{13}(\alpha), f_{14}(\alpha)\} \\ f_{11}(\alpha) = \dfrac{2\alpha + 8}{6\alpha + 6} \\ f_{12}(\alpha) = \dfrac{2\alpha + 8}{-6\alpha + 18} \\ f_{13}(\alpha) = \dfrac{-2\alpha + 12}{-6\alpha + 18} \\ f_{14}(\alpha) = \dfrac{-2\alpha + 12}{-6\alpha + 18} \qquad \text{where } 0 \le \alpha \le 1 \end{cases}$$

$$\begin{cases} \text{where } \max F_1(\alpha) = ? \\ \max F_1(\alpha) = \max\{ f_{11}(\alpha), f_{12}(\alpha), f_{13}(\alpha), f_{14}(\alpha)\} \\ f_{11}(\alpha) = \dfrac{2\alpha + 8}{6\alpha + 6} \\ f_{12}(\alpha) = \dfrac{2\alpha + 8}{-6\alpha + 18} \\ f_{13}(\alpha) = \dfrac{-2\alpha + 12}{-6\alpha + 18} \\ f_{14}(\alpha) = \dfrac{-2\alpha + 12}{-6\alpha + 18} \end{cases}$$

whose solutions are:

$$\min F_1(\alpha) = f_{11}(\alpha) = \frac{2\alpha + 8}{6\alpha + 6}$$

$$\max F_1(\alpha) = f_{14}(\alpha) = \frac{-2\alpha + 12}{-6\alpha + 18}$$

$$\therefore \tilde{\rho} = \left[\frac{2\alpha + 8}{6\alpha + 6}, \frac{-2\alpha + 12}{-6\alpha + 18}\right]$$

$$\tilde{\rho}^2 = [\min F_2(\alpha), \max F_2(\alpha)]$$

$$= \left[\frac{2\alpha + 8}{6\alpha + 6}.\frac{2\alpha + 8}{6\alpha + 6}, \frac{-2\alpha + 12}{-6\alpha + 18}.\frac{-2\alpha + 12}{-6\alpha + 18}\right]$$

where $0 \le \alpha \le 1$

$$\tilde{B} = 3\tilde{\rho}^2 + 4\tilde{\rho} + 2$$

$$= 3[\min F_2(\alpha), \max F_2(\alpha)] + 4[\min F_1(\alpha), \max F_1(\alpha)] + 2$$

$$= [\min F_3(\alpha), \max F_3(\alpha)] + [\min F_4(\alpha), \max F_4(\alpha)]$$

where $\min F_3(\alpha), \max F_3(\alpha), \min F_4(\alpha)$ and $\max F_4(\alpha)$ are solutions of PNLP as follows:

$$= \begin{cases} \text{where } \min F_3(\alpha) = ? \\ \min F_3(\alpha) = \min\{ f_{31}(\alpha), f_{32}(\alpha), f_{33}(\alpha), f_{34}(\alpha)\} \\ f_{31}(\alpha) = \dfrac{3.(2\alpha+8)^2}{(6\alpha+6)^2} \\ f_{32}(\alpha) = \dfrac{3.(-2\alpha+12)^2}{(-6\alpha+8)^2} \\ f_{33}(\alpha) = \dfrac{3(2\alpha+8)^2}{(6\alpha+6)^2} \\ f_{34}(\alpha) = \dfrac{3(-2\alpha+12)^2}{(-6\alpha+18)^2} \end{cases}$$

$$\begin{cases} \text{where } \max F_3(\alpha) = ? \\ \max F_3(\alpha) = \max\{ f_{31}(\alpha), f_{32}(\alpha), f_{33}(\alpha), f_{34}(\alpha)\} \\ f_{31}(\alpha) = \dfrac{3(2\alpha+8)^2}{(6\alpha+6)^2} \\ f_{32}(\alpha) = \dfrac{3(-2\alpha+12)^2}{(-6\alpha+18)^2} \\ f_{33}(\alpha) = \dfrac{3(2\alpha+8)^2}{(6\alpha+6)^2} \\ f_{34}(\alpha) = \dfrac{3(-2\alpha+12)^2}{(-6\alpha+18)^2} \end{cases}$$

$$= \begin{cases} \text{where } \min F_4(\alpha) = ? \\ \min F_4(\alpha) = \min\{ f_{41}(\alpha), f_{42}(\alpha), f_{43}(\alpha), f_{44}(\alpha)\} \\ f_{41}(\alpha) = 4.\dfrac{2\alpha+8}{6\alpha+6} \\ f_{42}(\alpha) = 4.\dfrac{-2\alpha+12}{-6\alpha+18} \\ f_{43}(\alpha) = 4.\dfrac{2\alpha+8}{6\alpha+6} \\ f_{44}(\alpha) = 4.\dfrac{-2\alpha+12}{-6\alpha+18} \end{cases}$$

$$= \begin{cases} \text{where } \max F_4(\alpha) = ? \\ \max F_4(\alpha) = \max\{ f_{41}(\alpha), f_{42}(\alpha), f_{43}(\alpha), f_{44}(\alpha)\} \\ f_{41}(\alpha) = 4.\dfrac{2\alpha+8}{6\alpha+6} \\ f_{42}(\alpha) = 4.\dfrac{-2\alpha+12}{-6\alpha+18} \\ f_{43}(\alpha) = 4.\dfrac{2\alpha+8}{6\alpha+6} \\ f_{44}(\alpha) = 4.\dfrac{-2\alpha+12}{-6\alpha+18} \end{cases}$$

whose solutions are:

$$\min F_3(\alpha) = f_{31}(\alpha) = 3.\frac{(2\alpha+8)^2}{(6\alpha+6)^2}$$

$$\max F_3(\alpha) = f_{34}(\alpha) = \frac{3(-2\alpha+12)^2}{(-6\alpha+18)^2}$$

$$\min F_4(\alpha) = f_{41}(\alpha) = 4.\frac{2\alpha+8}{6\alpha+6}$$

$$\max F_4(\alpha) = f_{44}(\alpha) = 4.\frac{-2\alpha+12}{-6\alpha+18}$$

Hence,

$$\tilde{B} = \left[\frac{132\alpha^2+480\alpha+456}{(6\alpha+6)^2}, \frac{132\alpha^2-1008\alpha+1944}{(18-6\alpha)^2} \right]$$

(i) Fuzzy Average Number of Production in the System

$$\tilde{L}_S^{\,1} = \frac{5\tilde{\rho}^2+4\tilde{\rho}}{\tilde{B}}$$

$$\tilde{L}_S^{\,1} = \frac{5[\min F_2(\alpha), \max F_2(\alpha)]+4[\min F_1(\alpha), \max F_1(\alpha)]}{3[\min F_2(\alpha), \max F_2(\alpha)]+4[\min F_1(\alpha), \max F_1(\alpha)]}$$

$$= \left[\frac{\dfrac{68\alpha^2+400\alpha+512}{(6\alpha+6)^2}, \dfrac{1584+68\alpha^2-672\alpha}{(18-6\alpha)^2}}{\dfrac{132\alpha^2+480\alpha+456}{(6\alpha+6)^2}, \dfrac{132\alpha^2-1008\alpha+1944}{(18-6\alpha)^2}} \right]$$

Applying interval arithmetic we obtain,

$$\tilde{L}_S^{\,1} = [\min F_5(\alpha), \max F_5(\alpha)]$$

$$\begin{cases} \text{where } \min F_5(a) \;=\; ? \\ \min F_5(\alpha) = \min\{ f_{51}(\alpha), f_{52}(\alpha), f_{53}(\alpha), f_{54}(\alpha)\} \\ f_{51}(\alpha) = \dfrac{68\alpha^2+400\alpha+512}{132\alpha^2+480\alpha+456} \\ f_{52}(\alpha) = \dfrac{(68\alpha^2+400\alpha+512)(18-6\alpha)^2}{(132\alpha^2-1008\alpha+1944)(6\alpha+6)^2} \\ f_{53}(\alpha) = \dfrac{(1584+68\alpha^2-672\alpha)(6\alpha+6)^2}{(132\alpha^2+480\alpha+456)(18-6\alpha)^2} \\ f_{54}(\alpha) = \dfrac{1584+68\alpha^2-672\alpha}{132\alpha^2-1008\alpha+1944} \end{cases}$$

$$\begin{cases} \text{where } \max \mathrm{F}_5(\mathrm{a}) \;=\; ? \\ \max F_5(\alpha) = \max\{ f_{51}(\alpha), f_{52}(\alpha), f_{53}(\alpha), f_{54}(\alpha)\} \\ f_{51}(\alpha) = \dfrac{68\alpha^2+400\alpha+512}{132\alpha^2+480\alpha+456} \\ f_{52}(\alpha) = \dfrac{(68\alpha^2+400\alpha+512)(18-6\alpha)^2}{(132\alpha^2-1008\alpha+1944)(6\alpha+6)^2} \\ f_{53}(\alpha) = \dfrac{(1584+68\alpha^2-672\alpha)(6\alpha+6)^2}{(132\alpha^2+480\alpha+456)(18-6\alpha)^2} \\ f_{54}(\alpha) = \dfrac{1584+68\alpha^2-672\alpha}{152\alpha^2-1008\alpha+1944} \end{cases}$$

whose solutions are

$$\min F_5(\alpha) = f_{51}(\alpha) = \frac{68\alpha^2 + 400\alpha + 512}{132\alpha^2 + 480\alpha + 456}$$

$$\max F_5(\alpha) = f_{54}(\alpha) = \frac{1584 + 68\alpha^2 + 672\alpha}{132\alpha^2 - 1008\alpha + 1944}$$

$$\tilde{L}_S{}^1 = \left[\frac{68\alpha^2 + 400\alpha + 512}{132\alpha^2 + 480\alpha + 456}, \quad \frac{68\alpha^2 - 672\alpha + 1584}{132\alpha^2 - 1008\alpha + 1944}\right]$$

(ii) Fuzzy Average Delay per Product by Fuzzy Little's Law is

$$\tilde{W}_S{}^1 = \frac{\tilde{L}_S{}^1}{\tilde{\lambda}_{eff}}$$

$$\tilde{p}_{00} = \frac{2}{\tilde{B}} = \left[\frac{72\alpha^2 + 144\alpha + 72}{132\alpha^2 + 480\alpha + 456}, \frac{72\alpha^2 - 432\alpha + 648}{132\alpha^2 - 1008\alpha + 1944}\right]$$

$$\tilde{p}_{01} = \frac{2\tilde{\rho}}{\tilde{B}} = \left[\frac{24\alpha^2 + 120\alpha + 96}{132\alpha^2 + 480\alpha + 456}, \frac{24\alpha^2 - 216\alpha + 432}{132\alpha^2 - 1008\alpha + 1944}\right]$$

$$\tilde{p}_{10} = \frac{\tilde{\rho}^2 + 2\tilde{\rho}}{\tilde{B}} = \left[\frac{28\alpha^2 + 152\alpha + 160}{132\alpha^2 + 480\alpha + 456}, \frac{28\alpha^2 - 264\alpha + 576}{132\alpha^2 - 1008\alpha + 1944}\right]$$

$$\tilde{p}_{11} = \frac{\tilde{\rho}^2}{\tilde{B}} = \left[\frac{4\alpha^2 + 32\alpha + 64}{132\alpha^2 + 480\alpha + 456}, \frac{2\alpha^2 - 48\alpha + 144}{132\alpha^2 - 1008\alpha + 1944}\right]$$

The probability that an arriving item will enter the station 1 in $\tilde{p}_{00} + \tilde{p}_{01}$

$$\tilde{p}_{00} + \tilde{p}_{01} = \left[\frac{96\alpha^2 + 264\alpha + 168}{132\alpha^2 + 480\alpha + 456}, \frac{96\alpha^2 - 648\alpha + 1080}{132\alpha^2 - 1008\alpha + 1944}\right]$$

(ii) Mean delay: $\tilde{W}_S{}^1 = \dfrac{\tilde{L}_S{}^1}{\tilde{\lambda}_{eff}}$

On simplification by interval arithmetic and PNLP we obtain as follows:

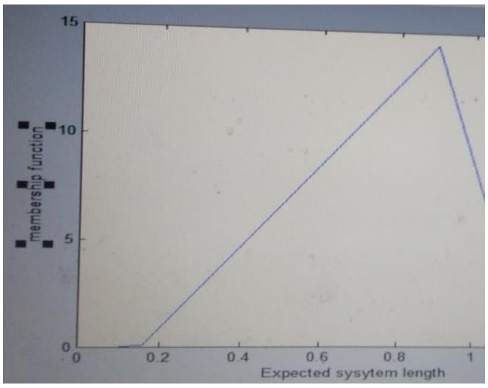
$$\tilde{W}_S{}^1 = \left[\frac{68\alpha^2 + 400\alpha + 512}{256\alpha^3 + 1920\alpha^2 + 4368\alpha + 3136}, \frac{68\alpha^2 - 672\alpha + 1584}{252\alpha^3 - 343\alpha^2 + 1520\alpha + 21600}\right]$$

If α runs from 0 to 1, the bounds of real intervals reveal the graphs of fuzzy tandem queue characteristics.

7. DISCUSSION

We explain the process consecutively by assigning α = 0 and α = 1 in $\tilde{L}_S{}'$, we attain $[\tilde{L}_S{}^1]_{\alpha=0} = [1.13, 0.82]$, $[\tilde{L}_S{}^1]_{\alpha=1} = [1,1]$. The range of $[\tilde{L}_S{}^1]_{\alpha=0}$ indicates that the expected fuzzy number of product in the system falls between 1.13 and 0.82. By numerical calculation, we capture insight into the possible average number of production and delay per unit product in the system.

α - cuts	$\tilde{L}_S{}'$
0	[1.13, 0.82]
0.1	[1.094, 0.88]
0.2	[1.07, 0.83]
0.3	[1.04, 0.84]
0.4	[1.02, 0.85]
0.5	[1, 0.86]
0.6	[0.981. 0.891]
0.7	[0.96, 0.88]
0.8	[0.947, 0.892]
0.9	[0.932, 0.904]
1.0	[1,1]



8. CONCLUSION

The penetration of this research work, is dealt with characteristics of fuzzy tandem queue with its steady state equations in 2 work-stations with PNLP by a new approach called 'flexible alpha-cuts method" based on the type of fuzzy arithmetic namely α-cuts and interval arithmetic. All arithmetic calculations are done on interval arithmetic which is non-fuzzy in nature. This method can be further extended to analysis of fuzzy queues with atmost 3 fuzzy parameters for tangibility, productivity and intuition in performance of future research work with stuporous and uncertain circumstances. Fuzzy tandem queuing models with flexible α-cuts method will have its wider exploration and extensibility for further findings.

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